HYDRODYNAMICS OF PLATE COLUMNS. IX.*
A DYNAMIC MODEL OF THE GAS–LIQUID MIXTURE ON A SIEVE PLATE WITHOUT DOWNCOMER

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Hinze's conclusions regarding the existence of oscillations of the gas–liquid mixture on a sieve plate relate also to a sieve plate without downcomer. In this paper the relation is examined between the frequency and the wave length of the oscillations by statistical analysis of random pressure fluctuations in the frequency domain. The frequencies appearing in the spectrum are associated with the standing-wave harmonic motion of the gas–liquid mixture, the resonant frequency of the air loop itself and the mechanism of bubble formation. The results in the amplitude domain point at a monotone increase of fluctuations of the pressure in the upper level of the plate and those of the pressure difference across the plate plus the froth with the increasing hold-up of liquid. In the region of homogeneous froth the found statistical characteristics can be used to divide the plate openings among those passed through by the liquid, the gas and those by-passed. The cross-correlation functions of the pressure fluctuations differ markedly under different regimes; the time-lag increases with the increasing hold-up of liquid. The calculation is given of the transfer of the fluctuations for a plate viewed as a "black box". A reliable generalization of the relations found requires additional experimental investigation.

The conclusion of more recent studies[1–5] dealing with the hydrodynamics of the plates without downcomers is that the factor controlling their operation, and to some extent also the operation of plates with downcomer, under a given regime is the distribution of the plate free area, *i.e.* the total number of openings or slots among those passed through by the liquid, by the gas and those by-passed. This implies the assumption of different pressure near the openings. Generally one can expect the pressure to vary with the position on the plate and the time as a consequence of various effects

Šteiner and Standart[1], for instance, give the difference of the "hydrostatic pressure", *i.e.* the difference of the clear liquid height over the openings occupied either by gas or liquid; this implies that all openings are passed through by either of the phases while the pressures above and below the plate are constant. Šteiner and Kolář[3] generalize the analysis to account for the existence of the by-passed openings and give the experimental verification for a plate without

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downcomer with no through-flow of liquid; in another paper[4] for a sieve plate with uniformly spaced downcomers.

A more general formulation of the problem can be found in the paper of Prince[5,6] analyzing the operation of a sieve plate in the region of weeping. Prince postulates that an opening switches the passing phases as a consequence of the dynamic fluctuations of the pressure drop across the plate in the given spot, which may be brought about by different height of liquid[1], or by dynamic fluctuations of pressure in the gas phase.

The analysis given in the cited paper was made on assumptions that *a*) all openings are passed by either of the phase, *b*) there is no interaction between the gas and liquid streams, and *c*) the dynamic fluctuations of pressure are the only cause.

The most realistic analysis of a two-phase gas–liquid flow on a plate in our opinion is that of Hinze[7]. Although it is devoted to the analysis of a plate with downcomer it may be essentially extended to a sieve plate without downcomer.

The main purpose of Hinze's analysis is a study of the conditions on the plate leading to either purely periodic, damped, or instable oscillations of the gas–liquid mixture. The analysis is carried out chiefly in the frequency domain without considering the importance of these fluctuations as a possible cause of alternating flow of the phases through the openings of a sieve plate with downcomers under the weeping regime. This regime is intimately associated with the operation of a sieve plate without downcomer, necessitating, however, analysis also in the amplitude domain. The work starts from a two-dimensional model of the gas–liquid mixture and a number of simplifying assumptions the most important of which are:

1) Constant time-averaged porosity of the gas–liquid mixture in the horizontal and vertical (x, y) direction, *i.e.* a homogeneous mixture.

2) The pressures below the plate, P_2, and above the mixture, P_3, are constant in space and time.

3) Linear distribution of the time-averaged static pressure over the height of the mixture: $\bar{P} - \bar{P}_3 = (\bar{P}_1 - \bar{P}_3)(1 - y/\bar{h})$.

4) The x and y component of the time-averaged velocity of liquid and the x component of the time-averaged velocity of gas equal zero.

5) The fluctuation components of both the vertical and horizontal velocity of liquid are functions of x and time; those of the vertical velocity of gas are functions of x, y, t and equal zero at $y = h$. The fluctuation components of the horizontal velocity of gas are identical to the corresponding component of liquid velocity. The fluctuation components of the height of the gas–liquid mixture are functions of y and t. The fluctuation components of the velocity consist of the oscillatory and the turbulent contribution, the latter having a substantially higher frequency.

6) The magnitude of the fluctuations is small making linearization permissible.

On linearization of the differential momentum and mass balances and on introducing a normalized disturbance of the height of the gas–liquid mixture as

$$\tilde{h}/\bar{h} = \text{konst.} \sin(2\pi x/\lambda) \,.\, \exp[rt(g/h)^{1/2}]\,, \qquad (1)$$

the author arrives at the following conclusions regarding the existence of the oscillations of the height of the gas–liquid mixture h:

a) For the dimensionless wave number, N, approaching zero, *i.e.* the wave length, λ, large in comparison with h, the instable oscillations of the frequency

$$\omega = (g/h)^{1/2} (\psi_{\mathrm{Hi}}^2 - 4\varphi_{\mathrm{Hi}}\mathrm{V})^{1/1}/2\varphi_{\mathrm{Hi}}\,, \qquad (2)$$

may occur if $\psi_{\mathrm{Hi}} < 4\varphi_{\mathrm{Hi}}\mathrm{F}$. The parameters defined by Hinze are given in the Appendix.

b) The values of the parameter φ_{Hi} approaching zero, pertaining to small free area plates of high dry-plate resistance give rise (neglecting the turbulent viscosity of liquid) to purely periodic oscillations *i.e.* the oscillatory regime of the frequency:

$$\omega = (g/h)^{1/2} \,.\, N[\psi_{Hi}{}^{+2}/(N^2 + 2\psi_{Hi})]^{1/2} \,. \tag{3}$$

If the turbulent viscosity is not negligible, the oscillations are damped. With increasing turbulent viscosity and the wave number the damping increases and the frequency ω decreases.

c) General solutions, that is those for the wave number ranging between 0 and ∞ and a finite dry-plate resistance ($\psi_{Hi} \neq 0$), can be obtained only in cases of negligible viscosity characterized by purely periodic oscillations $N_1 = \psi_{Hi}$, $N_2 = 2$ of the frequencies

$$\omega = (g/h)^{1/2} (\psi_{Hi})^{1/2} \,, \quad \omega = (2g/h)^{1/2} \,. \qquad (4a), (4b)$$

The oscillations are damped if $0 < N < \psi_{Hi}$. For non-zero turbulent viscosities the value of N_1 decreases with increasing φ and the increasing turbulent viscosity; N_2 increases.

Owing to non-linearity of the real process the instable oscillations are bounded by non-linear interactions.

The frequency of the purely periodic oscillations computed from Eq. (*3*) differs only little from the frequency of the harmonic standing gravitational waves[8] found by solving the equations of motion and the continuity for a potential flow of an inviscid liquid in a two-dimensional system of finite depth $\bar{h}$ and linearized boundary conditions on the freeboard:

$$\omega = [(2\pi g/\lambda) \tanh (2\pi\bar{h}/\lambda)]^{1/2} \,. \tag{5}$$

THEORETICAL

From Hinze's findings it follows that certain fluctuations (instable, purely periodic, or damped) of the height of the gas–liquid mixture on a sieve-plate always exist.

The model is formulated as a deterministic one, that is no parameter of the model possesses the character of a random variable. Existing dynamic studies of the gas–liquid mixture on a sieve plate[9,10] allow the process to be viewed as a quasi-stationary random process with distinctly random character particularly under the mobile froth regime. Nevertheless, one may assume that the characteristics of the random process in the frequency domain should reflect at least partially the essence of the relations derived by Hinze. The found agreement of the expressions for the purely periodic oscillations in Eqs (*3*) and (*5*) indicates that the dynamic behaviour of the gas–liquid mixture on a high resistance plate at sufficient gas velocities approximates the behaviour of the same depth of liquid in which the standing waves form as a consequence of the harmonic velocity potential. This suggests the importance of the two-dimensional solution to the wave equation for an inviscid liquid and the stochastic potential. Some additional considerations regarding the amplitude of these oscillations and their relation to the flow of the phases through the plate will be presented in the following.

Consider now a sieve plate without downcomer mounted between two chambers of equal cross-section, the lower of which serves as a gas supply. The flow rate of gas, G, supplied from the lower chamber and escaping from the upper one are both constant.

This physically simulates a plate built into a short loop of pipeline with a fan of constant volume pumping capacity.

For pressure fluctuations in the upper level of the plate under periodic oscillations we can write in accord with Hinze

$$\tilde{P}_1(x, t) = \tilde{P}_3(x, t) + A \sin \omega t \, . \sin (2\pi x/\lambda) \, . \tag{6}$$

For the spatially averaged pressure fluctuations in chambers P_2, P_3 then

$$-\frac{\mathrm{d}\bar{\tilde{P}}_2(t)}{\mathrm{d}t} = \frac{1}{C_2}(G_2 - \bar{G}) = \frac{1}{C_2} \varrho_G \bar{v}_G \left[\frac{\bar{\tilde{v}}_{G1}(t)}{\bar{v}_G} + \frac{\bar{\tilde{\varphi}}_G(t)}{\bar{\varphi}_G}\right] = \frac{1}{C_2} \varrho_G \bar{v}_G \frac{\bar{\tilde{v}}_{G2}}{\bar{v}_G} \, , \tag{7a}$$

$$\frac{\mathrm{d}\bar{\tilde{P}}_3(t)}{\mathrm{d}t} = \frac{1}{C_3}(G_3 - \bar{G}) = \frac{1}{C_3} \varrho_G v_G \frac{\bar{\tilde{v}}_{G3}}{\bar{v}_G} \, ; \tag{7b}$$

and for the pressure drop fluctuations in a position occupied by gas

$$\tilde{P}_2(x, t) - \tilde{P}_1(x, t) = K_G \varrho_G \frac{\bar{v}_G^2}{\bar{\varphi}_G^2} \left[\frac{\tilde{v}_{G2}(x, t)}{\bar{v}_G} - \frac{\tilde{\varphi}_G(x, t)}{\bar{\varphi}_G}\right] . \tag{8}$$

For their spatial average we have

$$\bar{\tilde{P}}_2(t) - \bar{\tilde{P}}_1(t) = K_G \varrho_G \frac{\bar{v}_G^2}{\bar{\varphi}_G^2} \left[\frac{\bar{\tilde{v}}_{G2}(t)}{\bar{v}_G} - \frac{\bar{\tilde{\varphi}}_G(t)}{\bar{\varphi}_G}\right] = 2 \, \Delta \bar{P}_{12} \left[\frac{\bar{\tilde{v}}_{G2}(t)}{\bar{v}_G} - \frac{\bar{\tilde{\varphi}}_G(t)}{\bar{\varphi}_G}\right] . \tag{9}$$

By combining Eqs (6)–(8) we get

$$K_G C_2 \frac{\bar{v}_{G1}}{\bar{\varphi}_G} \frac{\mathrm{d}\bar{\tilde{P}}_2(t)}{\mathrm{d}t} + \bar{\tilde{P}}_2(t) = \bar{\tilde{P}}_3(t) + \sin \omega t \int_0^L A \sin (2\pi x/\lambda) \, \mathrm{d}x - K_G \varrho_G \bar{v}_{G1}^2 \frac{\bar{\tilde{\varphi}}_G(t)}{\bar{\varphi}_G} \, . \tag{10}$$

Eq. (10) shows clearly the mutual relation of pressure fluctuations above and below the plate for the model considered.

For constant A one can show that

$$\int_0^L A \sin \omega t \sin (2\pi x/\lambda) \, \mathrm{d}x = 0 \, ,$$

where $L = n\lambda$ and n is a natural number; in all other instances the integral vanishes. Admitting though that the amplitude of the fluctuations A may be function of x (for simplicity *e.g.* in the form of a polynomial) as some papers seem to suggest[3,11], the integral will not vanish for all λ given by a polynomial of the first and higher degree. In this connection it should be mentioned that the observed standing harmonic oscillations in a finite volume of the gas–liquid mixture[12] lag in phase by $(\pi/2)$ in contrast to Eq. (*6*), that is

$$\tilde{P}_1(x, t) = \tilde{P}_3(x, t) + A \sin \omega t \cos (2\pi x/\lambda) \,. \tag{6a}$$

Then, of course, the value of the above integral at constant amplitude A equals zero for $L = n\lambda/2$; in all other cases remains non-vanishing.

For A expressed as a second or higher order polynomial in x the value of the integral is always non-vanishing. Eqs (*9*) and (*10*) express therefore the relation between the characteristic pressures for a linearized two-dimensional model of a sieve plate without downcomer.

Simultaneously one can judge from these equations on the relation between the statistical characteristics of these fluctuations in case of the random model, *i.e.* the variance, the autocorrelation and the cross-correlation functions and the spectral densities defined in detail in earlier papers[9,10].

From Eq. (*10*) one can infer that the spatial averaged pressure fluctuations below the plate are linear functions of the fluctuations above the mixture and another random input, determined mainly by the dynamic hold-up of liquid on the plate and the fluctuations in the passed through openings.

Predictably, the statistical estimates of the autocorrelation functions of individual fluctuations $P_i(t)$ will not be generally of a simple form. In case of no interaction between the individual inputs they equal the sum of the contributions of single inputs. Since the gas–liquid mixture on the plate must be necessarily regarded as a second-order system, one may expect that with random inputs[13]

$$r^{**}_{P_iP_i}(\tau) = \sum_i c_j \exp(-a_j) \left[2\pi b_j t + (a_j/2\pi b_j) \sin 2\pi b_j t\right], \qquad i = 1, 2, 3 \,. \tag{11}$$

The autocorrelation functions of this type satisfy the necessary condition of a continuous differentiable random process, *i.e.* $\mathrm{d}r^{**}_{PiPi}(0)/\mathrm{d}\tau = 0$.

It may be further expected that the strength of the linear relation $P_2 - P_3$, and hence the ratio of single inputs in Eq. (*10*), will differ markedly under various operating regimes of the plate and will effect the estimate of the coherence function

$$\gamma^{2*}_{P_2P_3}(f) = \frac{|G^{**}_{P_2P_3}(f)|^2}{G^{**}_{P_2P_2}(f)\, G^{**}_{P_3P_3}(f)}\,, \tag{12}$$

as well as the estimate of the normalized cross-correlation function (the correlation coefficient)

$$r^{**}_{P_2P_3}(\tau) = R^{**}_{P_2P_3}(\tau)/(D^{**}_{P_2P_2}(0)\, D^{**}_{P_3P_3}(0))^{1/2}\,. \tag{13}$$

The transformation of the pressure fluctuations in the amplitude domain and the shift in the time domain may be assessed from the estimate of the gain and the phase--lag of the transfer function

$$H(f)^{*}_{P_2P_3} = |H(f)^{*}| \exp\left(-j\Phi^{*}(f)\right), \tag{14}$$

where

$$|H(f)^{*}_{P_2P_3}| = \frac{|G^{**}_{P_2P_3}(f)|}{G^{**}_{P_2P_2}(f)} = \left[\frac{G^{**}_{P_3P_3}(f)}{G^{**}_{P_2P_2}(f)}\right]^{1/2}\,. \tag{15}$$

These parameters will probably depend strongly on the regime of the plate which affects the input ratio in Eq. (*10*).

Of the prime importance for a deeper study of the plate hydrodynamics aimed mainly at obtaining general correlations between controlling quantities and for the plate design are the statistical characteristics in the amplitude domain. In particular, the distribution and the standard deviation of the pressure differences which can be determined from a general relation

$$R^{**}_{\Delta P_{ij}}(\tau) = R^{**}_{P_iP_i}(\tau) + R^{**}_{P_jP_j}(\tau) - R^{**}_{P_iP_j}(\tau) - R^{**}_{P_jP_i}(\tau)\,. \tag{16}$$

TABLE I

Statistical Characteristics of the Fluctuation Components of Pressures and Pressure Differences under Various Operating Conditions

Experiment No	Regime	V_G (MS^{-1})	$\Delta\bar{P}_{23}$ (NM^{-2})	$z\,.\,10^3$ (M)	σ_{P_1}	σ_{P_2}	σ_{P_3} (NM^{-2})	$\sigma_{\Delta P_{12}}$	$\sigma_{\Delta P_{13}}$	$\sigma_{\Delta P_{23}}$
1	bubbling	0·15	70	4	17·2	26·7	12·6	27·8	20·6	20·3
2	cellular foam	0·35	160	11·0	30·6	11·7	8·9	37·5	39·0	9·0
3	mobile froth	0·60	400	33·2	53·9	17·6	13·4	61·5	60·1	17·8
4	oscillatory	1·82	940	71·2	109·8	41·6	25·9	135·2	120·0	46·6
5[a]	mobile froth	0·80	530	44·5	—	—	—	82·5	77·6	27·2
6[a]	mobile froth	1·00	680	58·6	—	—	—	89·3	93·0	29·5
7[a]	mobile froth	1·20	780	64·1	—	—	—	102·0	97·6	34·1

[a] Supplementary experiments designed to verify Eq. (*9*).

From Eq. (*9*) it follows for instance that on the given level of sophistication one can expect direct proportionality between the standard deviation of the fluctuation of the pressure drop across the plate on one hand and the standard deviation of the dimensionless expression $(\tilde{v}_{G2}/\bar{v}_G - \tilde{\varphi}_G/\bar{\varphi}_G)$ on the other hand.

EXPERIMENTAL

The experiments were carried out with the water–air system in a 300 mm in diameter column at various operating regimes of a sieve plate without downcomer. The free area of the plate was 15%, the opening diameter 6 mm.

The plate was built into a closed air loop with a radial fan. A detailed description of the experimental set-up, the experimental technique, data processing and the accuracy have been given in an earlier paper[10]. The experiments were carried out at the mass velocity of liquid equal 2·0 kg m^{-2} s^{-1} and the gas velocity ranging between 0·15 and 1·82 m s^{-1}.

RESULTS

Table I summarizes the operating conditions of the experiments together with the statistical characteristics of the fluctuation components of the pressures and their differences in the amplitude domain.

Figs 1 through 4 comprise the normalized autocorrelation and the cross-correlation functions from experiments 1 through 4.

Table II is a summary of the coefficients of the analytical expression in Eq. (*11*) of the normalized estimates of the autocorrelation functions $r^{**}_{P_1P_1}$.

The coefficients were obtained by a computerized non-linear regression due to Mar-

TABLE II

Explicit Expression of the Normalized Estimates of the Autocorrelation Functions

Exp. No	$r^{**}_{P_1P_1}$				$r^{**}_{P_2P_2}$				$r^{**}_{P_3P_3}$			
	j	c_j	a_j	b_j	j	c_j	a_j	b_j	j	c_j	a_j	b_j
1	1	—	—	—	1	0·60	0·11	10·49	1	0·14	0·11	10·53
	2	—	—	—	2	0·40	3·25	0·00	2	0·85	2·78	0·00
	3	—	—	—	—	—	—	—	—	—	—	—
2	1	0·75	0·75	4·25	1	0·56	3·13	21·90	1	0·90	0·46	0·37
	2	0·25	16·60	21·90	2	0·44	0·84	0·49	—	—	—	—
3	1	1·02	16·74	4·39	1	1·02	61·93	5·35	1	0·78	1·81	0·00
4	1	0·98	10·28	4·52	1	0·67	1·87	2·25	1	0·77	1·38	2·12

quardt[14]. The number of the components for individual autocorrelation functions varies both with single P_i and with the regime.

Table III comprises a similar information on the autocorrelation function $r^{**}_{\Delta P_{ij}}$ for the difference. The spectrum of the fluctuations is in Fig. 5.

TABLE III

The Autocorrelation Function of the Pressure Difference Fluctuations

Experiment No	$r^{**}_{\Delta P_{12}}$				$r^{**}_{\Delta P_{23}}$			
	j	c_j	a_j	b_j	j	c_j	a_j	b_j
1	—	—	—	—	1	0·79	0·11	10·54
2	—	—	—	—	1	0·78	3·83	22·06
3	1	0·88	9·60	4·53	—	—	—	—
4	1	0·78	4·49	2·73	1	0·98	2·88	2·32

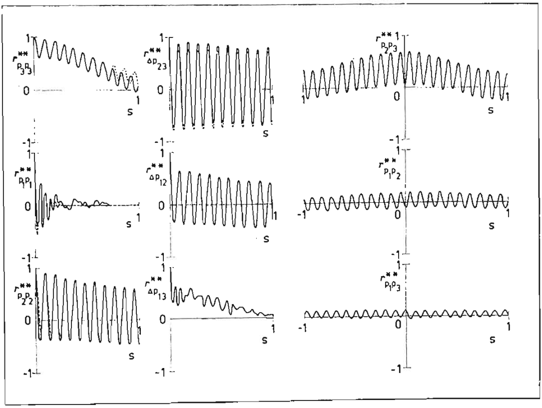

FIG. 1

Statistical Characteristics of Pressure Fluctuations in the Time Domain-Experiment No 1 ——— Experimental, – – – computed. Time τ given in s.

Table IV indicates the cross-correlation of individual fluctuations in the form of a maximum absolute value of the normalized cross-correlation function and the time-lag corresponding to the maximum. This Table together with Table V, summarizing the computed estimates of the coherence function for single frequencies (the coherence function was computed only for the frequencies whose normalized estimates

TABLE IV

The Cross-Correlation of Single Fluctuations

Experiment No	$\lvert r^{**}_{P_1P_2}\rvert_{MAX}$	$\lvert r^{**}_{P_1P_3}\rvert_{MAX}$	$\lvert r^{**}_{P_2P_3}\rvert_{MAX}$	τ_{12}	$\tau_{13} \cdot 10^3$ [s]	τ_{23}
1	0·21	0·13	0·73	37	— 10	— 47
2	0·17	0·22	0·62	1 415	—1 525	0
3	0·20	0·09	0·36	140	210	50
4	0·40	0·16	0·51	177	— 80	140

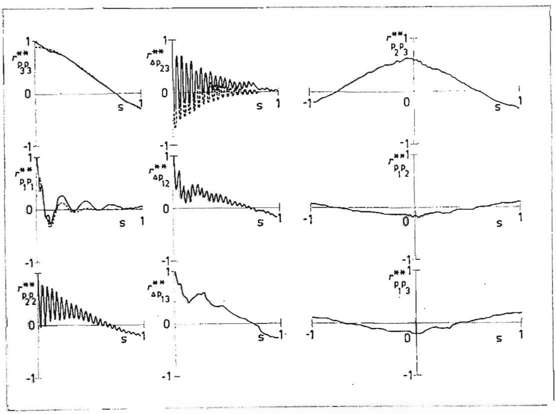

FIG. 2

Statistical Characteristics of Pressure Fluctuations in the Time Domain-Experiment No 2 ——— Experimental, – – – computed. Time τ given in s.

TABLE V

Computed Estimates of the Coherence Function for Single Frequencies

Experiment No	1		2		3		4	
f (Hz)	$\gamma^{2*}_{P_2P_3}(f)$	99% i.c.	$\gamma^{2*}_{P_2P_3}(f)$	99% i.c.	$\gamma^{2*}_{P_2P_3}(f)$	99% i.c.	$\gamma^{2*}_{P_2P_3}(f)$	9% i.c.
0·0	0·55	0·38—0·68	0·38	0·20—0·54	0·07	0·00—0·21	0·07	0·00—0·12
0·5	0·83	0·75—0·89	0·83	0·75—0·89	0·80	0·71—0·87	—	—
1·0	—	—	0·93	0·88—0·95	0·56	0·40—0·69	—	—
1·5	—	—	0·90	0·84—0·93	0·32	0·15—0·48	0·55	0·39—0·68
2·0	—	—	—	—	0·40	0·22—0·56	0·95	0·90—0·97
2·5	0·96	0·94—0·97	0·52	0·32—0·66	0·13	0·02—0·29	0·93	0·87—0·95
3·0	0·22	0·08—0·39	—	—	0·12	0·01—0·27	0·69	0·55—0·79
3·5	—	—	—	—	0·20	0·05—0·36	0·80	0·70—0·86
4·0	—	—	—	—	0·09	0·01—0·24	—	—
4·5	—	—	—	—	0·05	0·00—0·15	0·82	0·72—0·87
5·0	—	—	—	—	0·18	0·04—0·33	—	—
5·5	0·12	0·01—0·27	—	—	0·34	0·18—0·52	0·54	0·36—0·67
6·0	0·05	0·00—0·16	—	—	0·60	0·44—0·71	0·51	0·33—0·64
9·0	0·30	0·10—0·42	—	—	—	—	—	—
9·5	—	—	—	—	—	—	—	—
10·0	—	—	—	—	—	—	—	—
10·5	0·85	0·77—0·90	—	—	0·19	0·05—0·36	—	—
11·0	—	—	—	—	0·24	0·08—0·41	—	—

of the spectral density, $g^{**}_{P_2P_2}$, $g^{**}_{P_3P_3}$, were both greater than 0·001) and the 99% confidence intervals[13], evidence the different degree of linear cross-correlation of both pressures under various regimes and frequencies.

The frequency characteristics – the gain – plotted in Figs 6–9 point out by their different character on the differences in the mechanism of the transfer of pressure disturbances under various regimes. Aside from the gain computed from the known spectral densities $G^{**}_{P_2P_3}$ and $G^{**}_{P_2P_2}$, Figs 8 and 9 plot the values computed from $G^{**}_{P_2P_2}$ and $G^{**}_{P_3P_3}$. Thus we get a notion of the accuracy of the estimates which confirms the estimates of the accuracy made on the basis of the known correlation function[13]. The highest expected accuracy is that of the frequency characteristic in Fig. 9: in the range 1–10 Hz better than 20%, the worst accuracy for the characteristics in Fig. 8: in the range 1–10 Hz between 40–70%. The accuracy of the two remaining characteristics differs from point to point.

The verification of Eq. (9) is shown in Fig. 10. The values of $\Delta\bar{P}_{12}$ were obtained from

$$\Delta\bar{P}_{12} = \Delta\bar{P}_{23} - g\varrho_L Z\,, \tag{17}$$

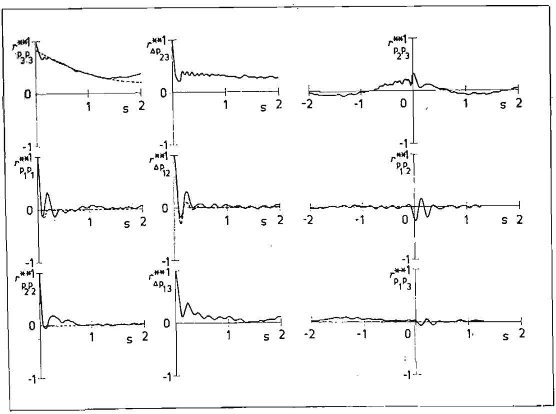

FIG. 3

Statistical Characteristics of Pressure Fluctuations in the Time Domain-Experiment No 3 ——— Experimental, – – – computed. Time τ given in s.

and from the relation

$$\Delta \bar{P}_{12} = \frac{1}{2\varphi_G^2} 2{\cdot}15 \varrho_G \bar{v}_G^2 \tag{18}$$

derived for the given type of plate by Červenka[11].

DISCUSSION

THE FREQUENCY DOMAIN

The various mechanisms of the passage of the gas phase through the plate reflect mainly in the type of the autocorrelation function, or the spectral densities.

It can be said that in the $\tilde{P}_1$ spectrum, which is directly associated with the oscillations of the height of the gas–liquid mixture, the peak appears between 4 and 5 Hz under all regimes. In experiments under the bubbling regime this component is accompanied by two more peaks around 10·5 and 21 Hz, under the cellular foam regime by a peak around 21 Hz, and under the oscillatory regime a peak around 2 Hz.

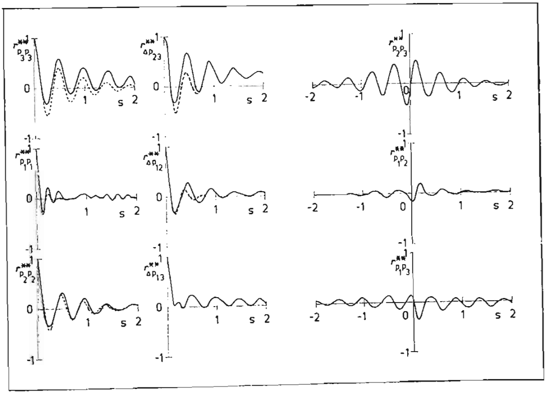

FIG. 4

Statistical Characteristics of Pressure Fluctuations in the Time Domain-Experiment No 4
——— Experimental, – – – computed. Time τ given in s.

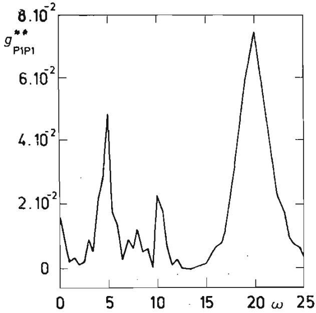

FIG. 5

The Spectrum of $\tilde{P}_1$ Fluctuations in Experiment No 1

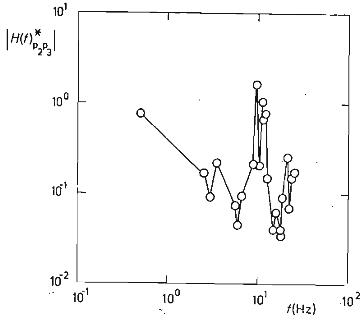

FIG. 6

Amplitude Frequency Characteristics for Experiment No 1

○ Values computed from $G^{**}_{P_2P_2}$ and $G^{**}_{P_2P_3}$.

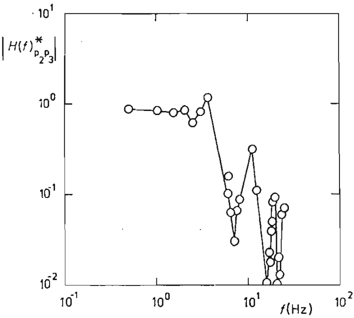

FIG. 7

Amplitude Frequency Characteristic for Experiment No 2

○ Values computed from $G^{**}_{P_2P_2}$ and $G^{**}_{P_2P_3}$.

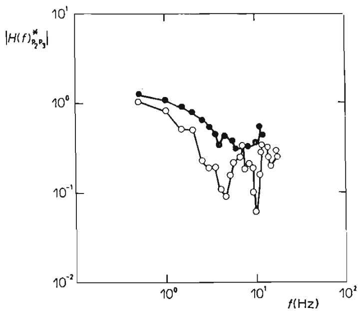

FIG. 8

Amplitude Frequency Characteristic for Experiment No 3

Values computed ○ from $G^{**}_{P_2P_2}$ and $G^{**}_{P_2P_3}$, ● $G^{**}_{P_2P_2}$ and $G^{**}_{P_3P_3}$.

Table VI gives the frequencies computed from Eq. (*3*) or (*5*) for the wave length $\lambda = D/4$ together with the frequencies of the corresponding component computed from the experimental data by the above mentioned method. The differences of the stochastic and the deterministic model, of course, has to be taken into account.

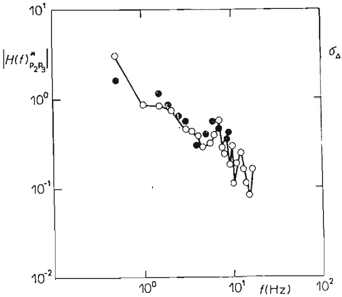

FIG. 9

Amplitude Frequency Characteristic for Experiment No 4

Values computed ○ from $G^{**}_{P_2P_2}$ and $G^{**}_{P_2P_3}$, ● $G^{**}_{P_2P_2}$ and $G^{**}_{P_3P_3}$.

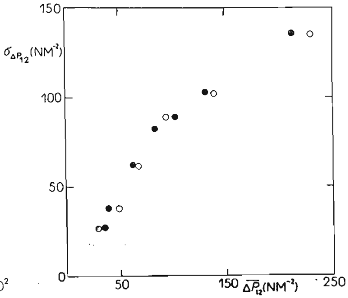

FIG. 10

A Comparison of the Experimental $\sigma_{\Delta P_{12}}$ with Eq. (*9*)

$\Delta \overline{P}_{12}$ computed ○ from Eq. (*17*), ● from Eq. (*18*).

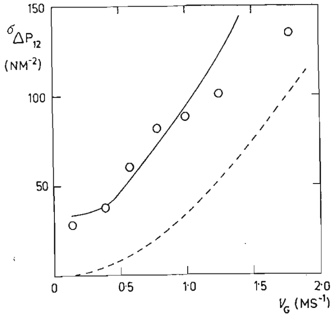

FIG. 11

A Comparison of the Experimental $\sigma_{\Delta P_{12}}$ with the Theoretical Relations

——— Eq. (*23*), – – – Eq. (*24*) according to Prince.

In accord with the model used for the calculation of the autocorrelation function[11], Eq. (*11*), we have

$$a_1 = 2\pi f_n \xi \,, \tag{19}$$

$$b_1 = 2\pi f_n (1 - \xi^2)^{1/2} \,, \tag{20}$$

where f_n is the resonance frequency of the system, ξ the coefficient of damping. From the values of b_1 found it follows that ξ ranges approximately between 5–16, the corresponding $(1 - \xi^2)^{1/2}$ values being only in the range 0·98–0·77. This indicates that even a marked change of ξ or a_1, which is immediately associated with the dynamic characteristics of the system, brings about only an inconspicuous change in the frequency spectrum, or in the shift of the corresponding peak with respect to the resonance frequency of the system.

As to the higher components found in experiments under the bubbling and the cellular foam regime it seems well founded to assume that these relate to the equations for bubble formation in single openings under conditions when the controlling term in the balance of forces is the surface tension[11]. On increasing the gas velocity from 0·15 to 0·35 m s^{-1} the average frequency of this "bubbling" component doubles and its relative presence in the spectrum diminishes to be replaced by the component with the peak around 4·5 Hz. The existence of the component with the peak around 21 Hz under the bubbling regime remains still unclear. This component though is not important for the mechanism of the flow through the plate openings and it does not appear in the fluctuations of $\tilde{P}_2$, $\tilde{P}_3$ or the pressure difference.

The existence of the 2·5 Hz component under the oscillatory regime of the gas–liquid mixture corresponds to the oscillations of the wave length $\lambda = D$, which have been observed under this regime. Visually they show up as a symmetric swelling of the level of the gas–liquid mixture near the column walls and in the center of the plate.

According to the simplified concept from the theoretical part there is no reason to expect the component corresponding to the oscillations of the gas–liquid mixture in the spectrums of the fluctuations of $\tilde{P}_2$, $\tilde{P}_3$ in case of the homogeneous mixture and under wave lengths satisfying $D = n\lambda/2$. From the presence of the 10 Hz component from $\tilde{P}_1$ spectrum appearing also in $\tilde{P}_2$ and $\tilde{P}_3$ spectrums under the bubbling regime one can infer that a large number of openings operates to an appreciable extent simultaneously, similarly as bubble formation in a single opening[6], causing a regular fluctuation of the gas flow rate through the plate. On transition to the cellular foam regime a similar 21 Hz component appears only in $\tilde{P}_2$ spectrum as a consequence of a stronger coupling between the openings and the space below the plate in contrast to the relaxed coupling with the space above the plate due to the forth formation. Nevertheless, the character of the corresponding component of the autocorrelation function points also at the randomized mechanism possibly by a random change of the number of the bubbling openings. In the spectrums of $\tilde{P}_2$ and $\tilde{P}_3$ under both regimes, and in $\tilde{P}_3$ spectrum under the mobile froth regime we find very low frequency components about 0·3 to 0·5 Hz.

In this limiting case $b_2 \to 0$ under the bubbling regime one can determine the resonance frequency from the corresponding autocorrelation function of $\tilde{P}_2, \tilde{P}_3$ as $a_2/2\pi$. Clearly, it is the resonance frequency of the whole air loop determined generally with the effect of inertia and the friction forces in the air tubing and the fan.

The simplified concept admits only capacity effects. By substituting the following values there result: $3{\cdot}25/2\pi$ or $2{\cdot}78/2\pi$, *i.e.* 0·519 or 0·442 Hz (the difference can be attributed to the expected statistical accuracy). The frequencies found in experiment 2 were 0·49 or 0·37 Hz; in experiment 3 in $\tilde{P}_3$ spectrum then the frequency $1{\cdot}82/2\pi$, *i.e.* 0·290 Hz.

Thus with the increasing velocity of gas the peak of this slow component shifts

TABLE VI

Computed Frequencies

Experiment No	φ_{Hi}	F_{Hi}	b_1	$\lambda = D/4$		$\lambda = D$	
				$\omega/2\pi$ from Eq. (3)	$\omega/2\pi$ from Eq. (5)	$\omega/2\pi$ from Eq. (3)	$\omega/2\pi$ from Eq. (5)
1	3·74	0·278	—	3·92	4·02	1·03	1·09
2	6·49	0·118	4·25	4·58	4·34	1·72	1·71
3	12·62	0·344	4·39	2·93	4·65	2·27	2·26
4	20·10	0·500	4·52	1·91	4·65	1·75	2·29

TABLE VII

Fractions of the Total Area of Plate Passed by the Phases and the By-Passed Area

Experiment No	Liquid $-\infty < \Delta P_{12} < -\Delta P_\sigma$	By-Passed $-\Delta P_\sigma < \Delta P_{12} < \Delta P_\sigma$	Gas $\Delta P_\sigma < \Delta P_{12} < \infty$
1	0·0244	0·3846	0·5910
2	0·0281	0·3966	0·5753
3	0·0548	0·1195	0·7257
5	0·0694	0·1664	0·7642
6	0·0808	0·1311	0·7881

toward low values. The relative presence of the slow component under the bubbling and the mobile froth regimes is approximately equal to that of higher components of $\tilde{P}_2$. On other hand, in $\tilde{P}_3$ spectrum the slow component is present in a greater degree.

Under the mobile froth and the oscillatory regime the $\tilde{P}_2$ spectrum contains components corresponding to the wave motion of the gas–liquid mixture. A feasible explanation rests in the non-homogeneity of the froth – see Eq. (*10*). The presence of the same component in the spectrum of $\tilde{P}_3$ under the oscillatory regime may be brought about by the same cause although its transfer from below the plate *via* air loop and fan cannot be ruled out.

In the spectrum of the pressure difference fluctuations, and hence in their autocorrelation functions computed from Eq. (*16*) as a linear combination of the corresponding auto- and cross-correlation functions, we find components from the spectrums of individual pressures in proportion to their amplitude characteristics and the magnitude of their cross-correlation. In this respect as the most interesting appears the spectrum of the pressure difference $\Delta\tilde{P}_{12}$ fluctuation associated directly with the mechanism of the gas flow through the plate openings.

Under the bubbling regime we find only the 10·5 Hz component in this spectrum; the other components do not appear.

The spectrum is most complex under the cellular foam regime and it comprises the components from the spectrum of $\tilde{P}_2$, *i.e.* the one around 0·4 Hz and both components of $\tilde{P}_1$ spectrum with the peaks around 4·5 Hz and 21 Hz. The mechanism of the gas flow through the plate openings thus combines the wave mechanism and that of the flow of single bubbles. Under the mobile froth regime, however, the only controlling factor of the gas flow through the plate is the wave motion of the gas–liquid mixture and in the spectrum of $\Delta\tilde{P}_{12}$ we find a single peak around 4·5 Hz.

The controlling effect under the oscillatory regime are the waves of $\lambda = D$, as it is evidenced by the presence of a single component around 2·3 Hz in the spectrum. In a similar way one can examine the spectrum of $\Delta\tilde{P}_{13}$ fluctuations in relation to the flow of the gas through the froth. Here too, one has to expect certain fluctuation of the height of the gas–liquid mixture to appear as a consequence of the wave motion. Under the conditions of our experiments, *i.e.* a large space above the gas–liquid mixture, the effect of the space between the plates is rather weak.

This becomes manifest particularly in the mobile froth and the oscillatory regime experiments: the 4·5 Hz component markedly prevails in the spectrum of the former, while in the latter regime we find a single component corresponding to $\lambda = D$.

In the experiment under the cellular foam regime the presence of the 4·5 and 0·4 Hz components points out to the effect of the waves in mixture as well as the flow oscillations in the air loop on the flow rate of gas through the froth. Similarly, under the bubbling regime it is the existence of the 10·5 Hz component and the one corresponding to the resonance frequency of the air loop.

In the spectrum of total pressure drop $\Delta\tilde{P}_{23}$ fluctuations one must expect the components of $\tilde{P}_2$ and $\tilde{P}_3$ spectrums.

Again, in the combination the amplitudes of individual components become important. As has been observed under the bubbling regime, there is only one (10·5 Hz) component present in the spectrum of $\Delta\tilde{P}_{23}$; the slow component 0·5 Hz has vanished altogether owing to several times greater amplitude of $\tilde{P}_2$ fluctuations and its weak representation in $\tilde{P}_2$ spectrum. A different situation occurs under the cellular foam regime. The approximately equal amplitudes of $\tilde{P}_2$ and $\tilde{P}_3$ fluctuations caused the 0·4 Hz and 21·5 Hz components to appear in the spectrum of $\Delta\tilde{P}_{23}$.

The autocorrelation function of $\Delta\tilde{P}_{23}$ fluctuations under the mobile froth regime suggests the presence of the slow component from the spectrum of $\tilde{P}_3$ fluctuations. The frequency corresponding to the wave motion of the gas–liquid mixture is almost undetectable owing to a largely random character of $\tilde{P}_2$ fluctuations and a weak $P_2 - P_3$ correlation. Under the oscillatory regime one finds only the component controlling the total pressure drop fluctuation of the frequencies induced by the wave motion of the gas–liquid mixture of $\lambda = D$.

The discussion of the results from the frequency domain may be summed up as follows: In the spectrum of the pressure and pressure difference fluctuations one finds components associated with the harmonic standing waves in the gas–liquid mixture together with the resonance frequency of the air loop and the mechanism of bubble formation in a single opening. A statistical analysis of the random data in the form of the spectral densities and the autocorrelation function enabled the examination to be made of the relations following from the deterministic model of Hinze.

Although the components with the peaks around 10·5 and 21 Hz could be related to the frequencies 2·3 and 4·5 Hz (which would correspond to $\lambda = D/16$ and $\lambda = D/64$) offering a relation between the distribution of the bubbling openings, plate geometry and the hold-up of liquid on the plate, such assertion requires more extensive experiments.

On the other hand, under certain conditions[11], the oscillations of the gas–liquid mixture of $\lambda = 2D$ do occur (unsymmetric swaying from one wall to the opposite) under which the frequencies of about (only an experimental average) 1·5 Hz (ref.[11]) were found which further supports the relations presented here between the wave length and the frequency.

The Cross-Correlation of Pressure Fluctuations; the Plate as a System with an Input and Output

From the maxima of the cross-correlation function of the pressure fluctuations $\tilde{P}_2$ and $\tilde{P}_3$ in Table IV one can infer that the strongest correlation between these fluctuations exists under the bubbling regime; the weakest under the mobile froth regime. Accordingly, the lowest values of the coherence function from Table V are those under the mobile froth regime.

Consider now these fluctuations as an input or an output of a system which in a first approximation is regarded as a "black box". In case of an ideal linear system with constant parameters, a single input and output and no noise in the input and the output, the coherence function in the whole frequency domain should equal unity[13]. In practice when the coherence function varies between 0 and 1 there are three explanations at hand. The presence of the noise in the input and/or output, the non-linearity of the system, and, finally, the system may be controlled by additional inputs and/or outputs.

Although the first two explanations are not implausible, it seems obvious already from Eq. (*10*) that the possible existence of other inputs, which may include noise, should be examined first. In case that these inputs are uncorrelated, one has to consider the frequency characteristics based on individual inputs.

On the given level of experimental knowledge it is difficult to asses the justification of such approach because the computed frequency characteristics, particularly those under the bubbling and the oscillatory regimes do not indicate its inadequacy. As to the time-lag summarized in Table IV, its relation to the mechanism can be succesfully interpreted only in experiment 1: The maximum increase of the pressure in the space above the plate caused by the flow of gas is followed after 10 ms by an increase of the pressure in the level of the plate accompanied apparently by weeping. This in turn is followed after 37 ms by an increase of the pressure below the plate accompanied by the bubble break-through and the flow of gas.

The explanation of the relation of the time-lag to the mechanism under other regimes requires additional experimental investigation accounting necessarily for the non-uniformity of the distribution of P_1 over the plate at higher gas velocities. One can conclude only that the time-lag between $\tilde{P}_2$ and $\tilde{P}_3$ fluctuations increases with the increasing height of the gas–liquid mixture.

To sum up one can say that the cross-correlation function, the correlation function and the computed frequency characteristics seem to indicate a direct relation to the mechanism of plate operation under various regimes. Quantitative conclusions, however, require additional experimental investigation.

THE AMPLITUDE DOMAIN

The results given in Table I as the standard deviations of individual pressure fluctuations point out unambiguously to the increase of the fluctuations in the level of the plate with increasing hold-up of liquid. Even here though one has to consider carefully the problem of having a sufficiently representative sample of $\tilde{P}_1$ fluctuations at high gas velocities, particularly under the oscillatory regime. This relation does not reflect in the pressure fluctuations above and below the plate.

The individual fluctuations under various regimes have different importance for the calculation of the standard deviation of the pressure differences $\Delta\tilde{P}_{12}$ and $\Delta\tilde{P}_{13}$ which control the operation of the plate. The effect of the fluctuations above and below the plate, however, cannot be generally neglected in this respect and particularly not so under the bubbling regime, *i.e.* under low gas velocities. The standard deviation of the pressure fluctuation across the plate openings and the froth also increases monotonously with the increasing velocity of gas and liquid hold-up. On the other hand this character does not reflect in the standard deviation of the fluctuation of the total pressure drop which has not been thus far explained.

From the $\sigma_{\Delta P_{12}}$ *versus* $\Delta\bar{P}_{12}$ plot in Fig. 10 one can see that the $\sigma_{\Delta P_{12}}/\Delta\bar{P}_{12}$ ratio has a constant value in the range of gas velocities where the assumption of homogeneous froth is justified. The average value of the ratio for $\Delta\bar{P}_{12}$ computed from Eq. (*18*) equals 0·92. For the values $\Delta\bar{P}_{12}$ computed from Eq. (*17*) this ratio amounts to 0·91.

Since the statistical distribution of the amplitude of the pressure fluctuations and their combinations may be justly regarded as normally distributed[9] (as follows for instance from the autocorrelation function) we find that the quantity $(\Delta P_{12} - \Delta\bar{P}_{12}) : : 0{\cdot}91\ \Delta\bar{P}_{12}$ is one with a normal distribution N(0, 1).

Considering now that the flow of gas in a given point of the plate can occur if $\Delta P_{12} > \Delta P_\sigma$, (where ΔP_σ is the pressure difference corresponding to the surface tension forces) and, on the contrary, the flow of liquid can occur only if $\Delta P_{12} < -\Delta P_\sigma$, one can determine from the knowledge of ΔP_σ the fraction of the time, θ, during which the plate in that point is by-passed by both phases. This time is then

$$\theta = [\text{i.n.p.d.f.}]_{(-\Delta P_\sigma - \Delta\bar{P}_{12})/0{\cdot}91\ \Delta\bar{P}_{12}}^{(\Delta P_\sigma - \Delta\bar{P}_{12})/0{\cdot}91\ \Delta\bar{P}_{12}} \; . \tag{21a}$$

The time when the plate is by-passed by the liquid phase is then

$$\theta = [\text{i.n.p.d.f.}]_{(\Delta P_\sigma - \Delta\bar{P}_{12})/0{\cdot}91\ \Delta\bar{P}_{12}}^{\infty} \; , \tag{21b}$$

or when it is by-passed by the gas phase

$$\theta = [\text{i.n.p.d.f.}]_{-\infty}^{(-\Delta P_\sigma - \Delta\bar{P}_{12})/0{\cdot}91\ \Delta\bar{P}_{12}} \; . \tag{21c}$$

The pressure difference necessary to overcome the surface tension forces can be calculated for a given type of plate from the equation derived by Červenka[11]

$$\Delta P_\sigma = 2{\cdot}35\sigma/d = 28{\cdot}6\ \mathrm{NM}^{-2}\,. \tag{22}$$

Clearly, this pressure difference is determined only by the properties of the liquid and the opening diameter. Thus it follows necessarily with increasing $\Delta \bar{P}_{12}$ the relative fraction of the time of by-pass is decreased. Table VII shows these times calculated from Eqs *(21)* for experiments in which the structure of the froth may be regarded as homogeneous in horizontal levels.

In such a case the local time distribution of the amplitude can be transformed to give the distribution over the area of the plate in a given instant and the fractions in Table VII can thus be regarded as those of the total free area of the plate passed-through by the gas, or liquid, or those by-passed by both phases.

Under the operating conditions, which exclude assumption of the homogeneous froth in horizontal levels (in our case at gas velocities greater than 1 m), the time distribution of the amplitude must be supplemented still by the distribution over the plate; the mentioned transformation then cannot be naturally utilized.

A comparison with the results of Prince and Chan[5] is furnished in Fig. 11 illustrating the agreement of experimental $\sigma_{\Delta P_{12}}$ with those computed from:

$$\sigma_{\Delta P_{12}} = 0{\cdot}91\bar{P}_{12} = 0{\cdot}91\left(\tfrac{1}{2}\varphi_G^2\right) 2{\cdot}15\varrho_G\bar{v}_G^2 \tag{23}$$

and with the theoretical equation

$$\sigma_{\Delta P_{12}} = \frac{1}{4\varphi^2}\left[\left(\frac{2{\cdot}78L^2}{\varrho_L}\right)^{1/3} + \left(\frac{1{\cdot}69G^2}{\varrho_G}\right)^{1/3}\right]^3 \tag{24}$$

following for the given plate from the cited paper.

From the amplitude distribution of the pressure fluctuations in the level of the plate one can clearly see the increase of the standard deviation with the hold-up of liquid. A similar character display also the fluctuations of the pressure drop across the plate and the froth.

The normal distribution of these fluctuations found permits their interpretation in relation to the division of the free area of the plate into that passed either by a single phase and that by-passed by both phases. The effect of the fluctuations above the froth and below the plate is not negligible and particularly not at low gas velocities.

For a limited set of experimental data the found semi-empirical dependence of the standard deviation on the velocity of gas in the region of homogeneous froth displays a substantially better fit than the theoretical relation of Prince and Chan[5].

APPENDIX

The definition equations of Hinze's parameters F_{Hi}, φ_{Hi}, ψ_{Hi} are as follows

$$F_{Hi} = (1 - \bar{\alpha})\, \bar{v}_G/(gh)^{1/2}\,, \tag{25}$$

$$\varphi_{Hi} = \frac{(1 - \bar{\alpha})^2\, \varphi^2}{\bar{\alpha}}\, \frac{\varrho_L}{\varrho_G}\, \frac{1}{K_G F}\,, \tag{26}$$

$$\psi_{Hi} = 2\left[1 - 2\,\frac{(1 - \varphi)^2}{K_G \bar{\alpha}}\right]. \tag{27}$$

LIST OF SYMBOLS

A	amplitude in Eq. (*6*) $(ML^{-1}\,T^{-2})$
C	acoustic capacity $(L^4\,M^{-1}\,T^2)$
D	column diameter (L)
D	variance
F_{Hi}	Froude number defined by Hinze, Eq. (*25*)
G	gas mass flow rate (MT^{-1})
G	spectral density
$H(f)$	transfer function
K	single-phase resistance coefficient of the plate
L	height of the plate (L)
L	liquid mass flow rate (MT^{-1})
$N = (2\pi/\lambda)\,\bar{h}$	wave number
P	pressure $(ML^{-1}\,T^{-2})$
R	autocorrelation function
a, b	constants in Eq. (*11*)
d	opening diameter (L)
f	frequency (T^{-1})
g	acceleration due to gravity (LT^{-2})
g	normalized spectral density
h	height of the gas-liquid mixture (L)
r	exponent in Eq. (*1*)
r	normalized correlation function
v	velocity (LT^{-1})
x	horizontal coordinate (L)
y	vertical coordinate (L)
t	time (T)
z	hold-up (L)
α	porosity of the gas–liquid mixture
$\gamma^2(f)$	coherence function
λ	wave length (L)
σ	surface tension (MT^{-1})
σ	standard deviation
τ	time-lag (T)
ξ	parameter in Eq. (*19*)
φ	plate free area

φ_{Hi}, ψ_{Hi} Hinze's parameters defined by Eqs (*26*, *27*)
$\omega = 2\pi f$ frequency (T^{-1})
θ fraction of time

Subscripts

1 in the level of the plate
2 below the plate
3 above the gas–liquid mixture
G gas
— average
~ fluctuation component
Δ difference
** statistical estimate

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